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An Introduction to Vedic Mathematics

Course Content

Indian Contribution to Mathematics

Sulva Sutra

The diagonal chord of the rectangle makes both the squares that the
horizontal and vertical sides make separately.
--Sulva Sutra
(8th century B.C.)

The *Sulva sutra* of the eighth century developed into the Pythagorean Theorem two hundred years later.

In the valley of the Indus River of India, the world's oldest civilization had developed its own system of mathematics. The Vedic Sulva Sutras (fifth to eighth century B.C), meaning "codes of the rope," show that the earliest geometrical and mathematical investigations among the Indians arose from certain requirements of their religious rituals. Vedic religious functions (Yajnas) required construction of several geometrical shapes which called for exact measurements, orientations, of different geometrical shapes. "Sulva Sutras" deals with the measurement and construction of altars and sacred places. The figures of the triangle, rectangle, circle and rhombus were a part of their use of plane geometry. The word sulva refers to the ropes used to make these measurements.

The square of the hypotenuse of a right angle triangle is
equal to the sum of the squares of the other two sides.
--Pythagorus Theorem
(6th century B.C.)

This theorem (540 B.C.), equating the square of the hypotenuse of a right angle triangle with the sum of the squares of the other two sides, was utilized in the earliest Sulva Sutra prior to the eighth century B.C. **Thus, widespread use of this famous mathematical theorem in India several centuries before its being popularized by Pythagorus has been documented**

When Greeks were groping in the Dark

A common belief among ancient cultures was that the laws of numbers have not only a practical meaning, but also a mystical or religious one. This belief was prevalent amongst the Pythagoreans. Prior to 500 B.C., Pythagorus, the great Greek pioneer in the teaching of mathematics, formed an exclusive club of young men to whom he imparted his superior mathematical knowledge. Each member was required to take an oath never to reveal this knowledge to an outsider. Pythagorus acquired many faithful disciples to whom he preached about the immortality of the soul and insisted on a life of renunciation. At the heart of the Pythagorean worldview was a unity of religious principles and mathematical propositions.

The Greeks, conquerors of the world however, encountered a major problem. The Greek alphabet, which had proved so useful in so many ways, proved to be a great hindrance in the art of calculating. Although Greek astronomers and astrologers used a sexagesimal place notation and a zero, the advantages of this usage were not fully appreciated and did not spread beyond their calculations. The Egyptians had no difficulty in representing large numbers, but the absence of any place value for their symbols so complicated their system that, for example, 23 symbols were needed to represent the number 986. Even the Romans, who succeeded the Greeks as masters of the Mediterranean world, and who are known as a nation of conquerors, could not conquer the art of calculating. This was a chore left to an abacus worked by a slave. No real progress in the art of calculating nor in science was made until help came from the East.

Arabic Numerals are not Arabic

A close investigation of the Vedic system of mathematics shows that it was much more advanced than the mathematical systems of the civilizations of the Nile or the Euphrates. The Vedic mathematicians had developed the decimal system of tens, hundreds, thousands, etc. where the remainder from one column of numbers is carried over to the next. The advantage of this system of nine number signs and a zero is that it allows for calculations to be easily made. Further, it has been said that the introduction of zero, or sunya as the Indians called it, in an operational sense as a definite part of a number system, marks one of the most important developments in the entire history of mathematics. The earliest preserved examples of the number system which is still in use today are found on several stone columns erected in India by **King Ashoka** in about 250 B.C.. Similar inscriptions are found in caves near Poona (100 B.C..) and Nasik (200 A.D.). These earliest Indian numerals appear in a script called *brahmi*.

After 700 A.D. another notation, called by the name "Indian numerals," which is said to have evolved from the brahmi numerals, assumed common usage, spreading to Arabia and from there around the world. When Arabic numerals (the name they had then become known by) came into common use throughout the Arabian empire, which extended from India to Spain, Europeans called them "Arabic notations," because they received them from the Arabians. However, the Arabians themselves called them "Indian figures" (Al-Arqan-Al-Hindu) and mathematics itself was called "the Indian art" (hindisat).

Mastery of this new mathematics allowed the Muslim mathematicians of Baghdad to fully utilize the geometrical treatises of **Euclid** and **Archimedes**. Trigonometry flourished there along with astronomy and geography. Later in history, Carl Friedrich **Gauss**, the "prince of mathematics," was said to have lamented that **Archimedes** in the third century B.C. had failed to foresee the Indian system of numeration; how much more advanced science would have been.

Prior to these revolutionary discoveries, other world civilizations-the Egyptians, the Babylonians, the Romans, and the Chinese-all used independent symbols for each row of counting beads on the abacus, each requiring its own set of multiplication or addition tables. So cumbersome were these systems that mathematics was virtually at a standstill. The new number system from the Indus Valley led a revolution in mathematics by setting it free. By 500 A.D. mathematicians of India had solved problems that baffled the world's greatest scholars of all time. **Aryabhatta**, an astronomer mathematician who flourished at the beginning of the 6th century, introduced sines and versed sines-a great improvement over the clumsy half-cords of Ptolemy.

The Zero

One of the commonest questions which arises in someone's mind is: Who discovered zero? Probably it is difficult to answer and we don't know which genius invented it. Most of the great discoveries and inventions of which Europe is so proud would have been impossible without a developed system of mathematics, and this in turn would have been impossible if Europe had been shackled by the unwieldy system of Roman numerals. The unknown man who devised the new system was, from the world's point of view, after the Buddha, the most important son of India. His achievement, though easily taken for granted, was the work of an analytical mind of the first order, and he deserves much more honour than he has so far received.

There are two uses of zero, which are both extremely important but are somewhat different. One use is as an empty place indicator in our place-value number system. Hence in a number like 2106 the zero is used so that the positions of the 2 and 1 are correct. Clearly 216 means something quite different. The second use of zero is as a number itself in the form we use it as 0. There are also different aspects of zero within these two uses, namely the concept, the notation, and the name.

Of course there are still signs of the problems caused by zero. Recently many people throughout the world celebrated the new millennium on 1 January 2000. Of course they celebrated the passing of only 1999 years since when the calendar was set up, no year zero was specified. Although one might forgive the original error, it is a little surprising that most people seemed unable to understand why the third millennium and the 21 century begin on 1 January 2001. Zero is still causing problems!

Indian Mathematicians

As early as the Vedic period (1500-1000BC), the Sulva Sutras facilitated the construction of sacrificial altars by their principles of plane geometry particularly through the figures of the triangle and the rectangle, the circle and the rhombus. Negative numbers, the zero, place-value notations and simple algorithms were already a part of mathematics. The great Indian mathematician Aryabhatta (born 476 AD) wrote the Aryabhatiya - a volume of 121 verses. Apart from discussing astronomy, he laid down procedures of arithmetic, geometry, algebra and trigonometry. He calculated Pi at 3.1416 and covered subjects like numerical squares and cube roots. Aryabhatta is credited with the emergence of trigonometry through sine functions.

The *Mahasiddhanta* of Aryabhata and the chapter *Lilavati* of *Bhaskaracharya's Siddhanta Siromani* discussed the value of pie as 22/7. The latter work was of four chapters-- the first: *Lilavati* (the beautiful) was on arithmetic. The second chapter: *Bijaganita* (root extraction) discussed algebra. The third and fourth chapters: *Ganitadhyaya* and *Goladhyaya* dealt with astronomy. The text included a technique of very speedy calculation not unlike modern calculus.

Medieval Indian mathematicians, such as Brahmagupta (seventh century), Mahavira (ninth century), and Bhaskara (twelfth century), made several discoveries which in Europe were not known until the Renaissance or later. They understood the importance of positive and negative quantities, evolved sound systems of extracting square and cube roots, and could solve quadratic and certain types of indeterminate equations." Mahavira's most noteworthy contribution is his treatment of fractions for the first time and his rule for dividing one fraction by another, which did not appear in Europe until the 16th century.

Around the beginning of the sixteenth century Madhava developed his own system of calculus based on his knowledge of trigonometry. He was an untutored mathematician from Kerala, and preceded Newton and Liebnitz by more than a century.

The twentieth century genius Srinivas Ramanujan (1887-1920) developed a formula for partitioning any natural number, expressing an integer as the sum of squares, cubes, or higher power of a few integers.

Vedic Mathematics an Overview

The "Vedic Mathematics" is called so because of its origin from Vedas. To be more specific, it has originated from "Atharva Vedas" the fourth Veda. "Atharva Veda" deals with the branches like Engineering, Mathematics, sculpture, Medicine, and all other sciences with which we are today aware of.

This wonderful method is re-introduced to the world by Swami Bharati Krisna Tirtha ji Mahaharaj, Shankaracharya of Goverdhan Peath. "Vedic Mathematics" was the name given by him. He was the person who collected lost formulae from the writings of "Atharwa Vedas" and wrote them in the form of Sixteen Sutras and thirteen sub-sutras. (Sutras, subsutras and their English meaning has been given in the origin section of our website. http://www.magicalmethods.com)

How Vedic Mathematics is different from Conventional System?

Each system has got its own limitations. Similarly the **Conventional System** (which you are using at present) has its limitations too. IF you do a lot of practice then also you will not be able to improve your calculation speed beyond a certain point. Why?

The answer is the System that you are using has reached its limitations.

But surprisingly, People take it as their limitation.

Here we are putting a comparison between **Conventional Method** and **Magical Method** for you to have a look.

Conventional Method	Magical Method
28232	28232
× 53246	× 53246
169392	**1 5 0 3 2 4 1 0 7 2**
112928	5 4 5 7 7 3 2 1
56464	
84696	
141160	
1503241072	

This has been done using **Criss-cross** technique of Vedic Mathematics.

Once you have little practice you can do it straight: 28232 × 53246 = 1503241072

Vedic Mathematics introduces the wonderful applications to Arithmetical computations, theory of numbers, compound multiplications, algebraic operations, factorisations, simple quadratic and higher order equations, simultaneous quadratic equations, partial fractions, calculus, squaring, cubing, square root, cube root , coordinate geometry and the wonderful Vedic Numerical code.

Uses of Vedic Mathematics:

- It helps a person to solve mathematical problems 10-15 times faster.
- It helps in Intelligent Guessing
- It reduces burden (need to learn tables up to 9 only)
- It is a magical tool to reduce scratch work and finger counting
- It increases concentration.
- It helps in reducing silly mistakes

Vedic Mathematics Sutras

S. No.	Sutras	Meaning
1.	*Ekadhikena Purvena*	One more than the previous
2.	*Nikhilam Navatascharamam Dastah*	All from nine and last from ten
3.	*Urdhwa-tiryagbhyam*	Criss-cross
4.	*Paravartya Yojayet*	Transpose and adjust
5.	*Sunyam Samyasamuchchaye*	When the samuchchaya is the same, the samuchchaya is zero, i.e it should be equated to zero.
6.	*(Anurupye) Sunyamanyat*	If one is in ratio, the other one is zero.
7.	*Sankalana-vyavkalanabhyam*	By addition and by subtraction
8.	*Puranpuranabhyam*	By completion or non-completion
9.	*Chalana-Kalanabhyam*	Differential
10.	*Yavdunam*	Double
11.	*Vyastisamastih*	Use the average
12.	*Sesanyankena Charmena*	The remainders by the last digit
13.	*Sopantyadyaymantyam*	The ultimate & twice the penultimate
14.	*Ekanyunena Purvena*	One less than the previous
15.	*Gunitasamuchachayah*	The product of the sum of coefficients in the factors
16.	*Gunaksamuchchayah*	When a quadratic expression is product of the binomials then its first differential is sum of the two factors

Vedic Mathematics sub-Sutras

S. No.	Sutras	Meaning
1.	*Anurupyena*	Proportionately
2.	*Sisyate Sesasamjnah*	Remainder remains constant
3.	*Adyamadyenantyamantyena*	First by first and last by last
4.	*Kevalaih Saptakam Gunyat*	In case of seven our multiplicand should be 143
5.	*Vestanam*	Osculation
6.	*Yavdunam Tavdunam*	Whatever the extent of its deficiency, lessen it still further to that very extent
7.	*Yavdunam Tavdunam Varganchya Yojayet*	Whatever the extent of its deficiency, lessen it still further to that very extent; and also set up the square of that deficiency.
8.	*Antyayordasakepi*	Whose last digits together total 10 and whose previous part is exactly the same
9.	*Antyayoreva*	Only the last terms
10.	*Samuchchyagunitah*	The sum of the coefficients in the product
11.	*Lopanasthapanabhyam*	By alternate elimination and retention
12.	*Vilokanam*	By observation
13.	*Gunitsamuchchayah Samuchchayagunitah*	The product of sum of the coefficients in the factors is equal to the sum of the coefficients in the product.

About Creator of Vedic Mathematics

Swami Bharati Krisna Tirtha Ji Maharaja, the author of **Vedic Mathematics** was the Sankaracharya (Pontiff) of the Govardhan Math, Puri from 1925 to 1960 AD. His Holiness Sri Bharati Krisna Tirtha was born in March 1884. His parents named him as Venkatraman. Even during his school days Sri Venkatraman was extraordinarily bright. Looking at his proficiency in Sanskrit, At such an early age of fifteen, he was awarded the title **Saraswathi** by Madras Sanskrit Association in 1899. Just at the age of twenty Sri Venkatraman Saraswati passed M.A. examination securing the highest honors in all subjects.

He passed his M.A. Examination of the American College of Science, Rochester, New York, with Sanskrit, Philosophy, English, Mathematics, History and Science, securing the highest honours in all the seven subjects. As Professor Venkatraman Saraswati, he started his public life under the guidance of Late Hon'ble Shri Gopal Krishna Gokhale, C.I.E. in 1905 in connection with the National Education Movement and the South African issue.

Right from the childhood Sri Venkatraman was spritually inclined. Due to this he proceeded, in 1908, to Sringeri Math in Mysore to lay himself at the feet of the renowned late Jagadguru Shankaracharya Maharaj Shri Satchidananda Shivabhinava Nrisimha Bharati Swami. In 1908 or so he got the post of the first Principal of the newly started National College at Rajmahendri under a pressing and clamant call of duty from the nationalist leaders. But after three years, in 1911, he went back to Shri Satchidananda Shivabhainava Nrisimha Bharati Swami at Sringeri.

The next eight years he spent in the profoundest study of the most advanced Vedanta Philosophy and practice of the Brahma-sadhana. In 1919 he was initiated into the holy order of Samnyasa at Varanasi by H. H. Jagadguru Shankaracharya Shri Trivikrama Tirthaji Maharaja of Sharadapeetha and was given the new name, Swami Bharati Krishna Tirtha. And in 1921, he was installed on the pontifical throne of Sharda Peetha Shankaracharya, and in 1925 he shifted to

Puri when he was installed as Jagadguru Shankaracharya of the Govardhan Math, while Shri Swarupanandaji was installed on the Shardapeetha Gadi. In 1953, he founded at Nagpur an institution named Shri Vishwa Punarnirmana Sangha (World Reconstruction Association), with Shri Chimanlal Trivedi as the General Secretary and the Administrative Board consisted of his disciples, devotees and admirers.

Thus, it seems he discovered the VM Sutras during his stay at Shringeri, between 1911 and 1919, and at the age of his 34th or 35th year and for next few years he was busy working on these Sutras, and he seems to have definitely written, in school notebooks, all of his sixteenth volumes treating each of his sixteen Sutras in one independent volume, most probably well before 1953.

Shri Chimanlal Trivedi and Smt. Manjulaben Trivedi, originally belonged to Kapadwanj (District Kheda, Gujarat), and perhaps they turned into devotees of BKTM, when as the Jagadguru Shankaracharya visited Kapadwanj for the monsoon months of the year 1937 or so. During his stay at Kapadwanj, BKTM is said to have borrowed some amount from Shri Manilal Desai, a money-lender of Dakor and devotee of him, and as a security against the borrowed amount, he had pledged his manuscript notebooks safely stuffed and packed in a few tin trunk boxes. It seems, BKTM somehow could not repay the amount to Shri Manilal Desai till the end of the latter's lifetime, say till about the year 1955 or so. And then, the boxes came under the charge of Shri Laxminarayan, the son of Shri Manilal Desai. Shri Laxminarayan got them transported from Dakor to his residence at Asarva in Ahmedabad.

Professor Vijaya M. Sane, who had a great interest in tracing the location of the boxes containing the manuscript note-books of the VM in sixteen volumes, took great pains in utilising his contacts with the higher up in the Gujarat Government and Gujarat Police Department, and finally located the residence of Shri Laxminarayan in Ahmedabad, got in touch with Jagadguru Shankaracharya Swami Shri Abhinava Sachidananda Tirthaji Maharaj of Sharada Peetha, Dwarka (Gujarat), got the boxes confiscated and searched. But the boxes were found to contain useless scraps and old shoes and such other trash. After that when Prof. Sane contacted Shri Laxminarayan personally and talked to him, he was told that some German scholar had came searching for him and had offered big amount for the VM manuscript material of the sixteen volumes; that he had sold the contents of the boxes to that German scholar for Rs. 80,000/-, and had stuffed the boxes with rubbish. Even after that Shri Sane tried his utmost to trace the whereabouts and identity of the German scholar, with the help of Hon'ble Shri Hitendrabhai Desai, Chief Minister of Gujarat at that time, but his efforts donot seem to have met with any degree of success so far. This must have happened some time in the year 1955-56, since, "unfortunately, the said manuscripts were lost irretrievably from the place of their deposit and this colossal loss was finally confirmed in 1956".

After the confirmation that all the volumes were lost in 1956, Swamiji was sincerely requested by his ardent devotees to make them available again by rewriting them. On their request, it seems, it was finally in 1957, when he decided to undertake a tour of the U.S.A., swamiji wrote down the volume in his old age within one month and a half with his failing health and weak eyesight. The type-script of the VM was left over by BKTM in U.S.A. in 1958 for publication. In 1960, Swamiji attained Maha samadhi and the book **vedic mathematics** by him got published in 1965.

Thus, it seems to be a solid fact that BKTM did write down his sixteen volumes on the sixteen Sutras of the Vedic Mathematics, that he deposited his manuscript with his devotee Shri Manilal Desai of Dakor in Gujarat, that after the death of Shri Manilal, the material came in possession of his son Laxminarayan Desai, and the latter sold it to some German scholar for Rs. 80,000/-, and that, at least subconsciously, BKTM was under the impression that he was writing the book overall again, in a series of a number of volumes, although what he could write was a single volume published as the VM.

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About Magical Methods

Mr. Pradeep Kumar and his team founded Magical Methods to spread "Vedic Mathematics" through out the world. The way it is growing makes us more than hopeful that this dream will come true one day.

Founder Members

Pradeep Kumar [B. Tech, MBA IIMB]: More than 15 years experience in industry. Magical Methods is his brain child. He is a researcher to the core and a remarkable trainer. He owns more than 150 copyrights and 20 patents.

Vinayak Joshi[MBA IIMB]: More than 12 years experience in industry. A Finance professional to the core of his heart. Great believer in the idea called Vedic Mathematics. His prudent financial management has taken Magical Methods to new heights.

Sanjay Chaudhary [B. Tech, MBA IIMB]: More than 15 years experience in industry. A Marketing professional and a solid strategist. His marketing strategy has started bearing fruits now.

Rajiv Buddhiraja [B. Tech, MBA IIML]:More than 12 years experience in industry. A person who can manage any complex production system. Magical Methods is gaining a lot from his expertise.

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Growth of Magical Methods

May 2000: started www.magicalmethods.com

2000-2002: Established www.magicalmethods.com as a leading website in the field of Vedic Mathematics. In the process a lot of online courses got created. Now we are running online courses for General users to specialized users like GMAT and CAT aspirants. We also train teachers in Vedic Mathematics online.

2003: Opened first offline centre to teach Vedic Mathematics in Gurgaon.

2004: Opened first training centre in Delhi for teaching Vedic Mathematics. Subsequently two other centers were opened in Delhi.

2005: Opened first training centre in Dubai (UAE) for teaching Vedic Mathematics.

2006: First Master Franchise given out in Gujarat (India). Second Master Franchise given out in Dubai (UAE).

2007: Opened a Training centre in Kathmanu (Nepal). Presently we run more than 50 training centers in India and abroad. For details please visit **Contact us** link on our website.

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Our Programmes

Kid Segment

Class 1 to 6 (age: 8 yrs. To 12 yrs.) (**Finger Counting & Silly Mistakes Removal Programme**)

We offer classes once or twice a week.

Duration: 12 Months/ 24 Months. [(24 months if classes are offered once a week), 4 Levels, Duration of each Level: 24 classes of one and half hrs each, Total 36 Hrs. of class]

Teen Segment

Class 7 to 12 (13 yrs and above) (**Vedic Mathematics : Beginner**)

Duration: One and Half Month (classes twice a week, 24 classes of one and half hrs each, Total 36 Hrs. of class)

Class 7 to 12 (13 yrs and above) (**Vedic Mathematics : Medium**)

Duration: One and Half Month (classes twice a week, 24 classes of one and half hrs each, Total 36 Hrs. of class)

Class 7 to 12 (13 yrs and above) (**Vedic Mathematics : Expert-1**)

Duration: One and Half Month (classes twice a week, 24 classes of one and half hrs each, Total 36 Hrs. of class)

Students are provided with Study Material (Books etc.), (Charges apply)

Course Fee may vary depending upon the Market.

Introduction to First Formula

Objective-1:

1. How to find out square of a two-digit number ending with 5?

Let us start with an example: -

45
× 45

How would you multiply this in conventional way?

45
× 45
2 2 5
1 8 0
2 0 2 5

What are the steps you took here?

1. First you multiplied 45 by 5 and wrote it below the line (225).
2. Then you multiplied 45 by 4 and wrote it below the first row leaving one space from right (180).
3. You added the numbers in first row with the numbers in the second row by first putting right most digit down and adding other digits thereafter conventionally.
4. You got 2025 as answer.

Now let us do it by magical method: -

45
× 45
2025

What did we do here?

1. We multiplied 5 by 5 and put 25 as right hand side of the answer.
2. We added 1 to the top left digit 4 to make it 5.

3. We then multiplied it (5) by bottom left digit 4 and got 20, this is left hand side of the answer.
4. We arrived at our desired answer 2025.

Did you get it?

Let us do some more by the method learned just now!

75
× 75
5625

Let me explain the method again !!

1. We multiplied 5 by 5 and put 25 on the right hand side.
2. We added 1 to the top left digit 7 to make it 8.
3. We then multiplied 8 by bottom left digit 7 and kept 56 on left-hand side.
4. We arrived at our desired answer 5625.

Now the method should be crystal clear to you.

In the same manner, we can multiply the following: -

15 by 15, 25 by 25, 35 by 35, 55 by 55, etc.

Exercise-1

1.	75 × 75	2.	35 × 35	3.	85 × 85	4.	95 × 95

5.	25 × 25	6.	55 × 55	7.	15 × 15	8.	65 × 65

Objective-2:

2. How to find out square of a three-digit number ending with 5?

Let us start with an example: -

125
× 125

What did we do here?

5. We multiplied 5 by 5 and put 25 as right hand side of the answer.
6. We added 1 to the top left digit 12 to make it 13.
7. We then multiplied it (13) by bottom left digit 12 and got 156, this is left hand side of the answer.
8. We arrived at our desired answer 15625.

Things to be kept in mind: In two digit multiplied by two digit number after multiplying 5 by 5 there were only one digit on the left hand side. Whereas, in this case there are two digits on the left hand side therefore you would add one to first two digits for the purpose.

Exercise-2

1.	105 × 105	2.	115 × 115	3.	135 × 135	4.	155 × 155

Objective-3:

Explanation of the sutra

I understand, you are getting inquisitive here and planning to ask a loaded question.
Your question is whether the applicability of the formula is limited to a number ending with 5 only?
My answer is no, its not like that.
Let us expand the formula……………….
We can apply this formula to find multiplication of a good amount of two digit, three digit numbers.

Preconditions are: -

Left-hand digits should be same and addition of right hand digits should be *10*.

Let us take an example: -

66
× 64
4224

In this example left-hand digits are same i.e. 6 and addition of right-hand digits are 10. So we can apply this formula here.

Can we apply the same formula to the following: -

1.	67 × 63	2.	48 × 42	3.	86 × 84	4.	71 × 79
	4221		2016		7224		5609

Yes, We can apply the same formula to all these since *their* left-hand digits are same and addition or right hand digits are 10.
Here another question may creep into your mind that in the third one above when 9 is multiplied by 1 then it gives 9, but how come we are putting 09 there. The answer is simple, from all above examples we have learned that the right hand side should have two digits but we are getting only one digit i.e. 9 so what to do? How can we use this harmlessly without changing its value? You know it, add 0 to the left. Now see whether your formula is applicable to the given examples: -

1.	46 × 44	2.	47 × 43	3.	48 × 42	4.	49 × 41

I know that, your answer is affirmative and you can write the answers as 2024, 2021, 2016 and 2009.

Exercise-3

1.	26 × 24	2.	33 × 37	3.	82 × 88	4.	96 × 94
5.	63 × 67	6.	52 × 58	7.	68 × 62	8.	34 × 36
9.	38 × 32	10.	64 × 66	11.	91 × 99	12.	83 × 87

Introduction to Quick Formula

YAVDUNAM

This sutra function over a base value. The bases may be 10, multiples of 10 or 100, multiples of 100 or 1000, multiples of 1000.

For this discussion we will limit ourselves to base 10, multiples of 10 and 100, multiples of 100.

Uses of Yavdunam Sutra for finding square of a number near 100.

Say you want to find square of a number near 100 or multiples of 100. First we will discuss how to find square of a number which is near 100.

Let the number be 98.

In conventional method, you will be required to multiply 98 by 98 as shown below: -

98
× 98
784
882
9604

The answer comes to 9604. But it is a good deal of work.

Are you interested in something really fast? Faster than computer then learn this formula.

The formula says whatever be the difference of the number from the base add (if the number is more than the base) or subtract (if the number is less than the base) that much to the number and on the right hand side set the square of the difference. This is your answer.

Let us start working from base 100 (numbers more than 100).

Say you want to find out 106^2

106^2 = $106+6/6^2$ = 11236

Kind in mind, since 106 is 6 more than 100 therefore we have added 6 here as per the Sutra and on the right hand side we set square of 6.

Solved Examples: -

1. 107^2 = $107+7/7^2$ = 114/49 = 11449
2. 109^2 = $109+9/9^2$ = 118/81 = 11881

Please note that the number of digits on the Right hand Side would be equal to number of zeros in our base. In this case our base is 100 where number of zeros is equal to 2. Therefore, number of digits on the right hand side would be equal to 2. **It means that when there is only one digit on the right hand side you would make it 2 by adding a zero. e.g.**

1. 103^2 = $103+3/3^2$ = 106/9 = 10609

Also when there are three digits on the right hand side you would make it 2 by carrying one digit to left hand side. e.g.

1. 111^2 = $111+11/11^2$ = 122 / <u>**1**</u>21 = 12321
2. 116^2 = $116+16/16^2$ = 132 / <u>**2**</u>56 = 13456

Exercise - 4

1.	101^2	=	
2.	104^2	=	
3.	102^2	=	
4.	108^2	=	
5.	112^2	=	
6.	115^2	=	
7.	113^2	=	
8.	121^2	=	

9.	123^2	=	
10.	124^2	=	

Base 100 (numbers less than 100).

Say you want to find out 98^2

98^2 = $98 - 2/ 2^2$ = 96/4 = 9604

98 is 2 away (less) from 100 therefore 2 is reduced from 98 in L.H.S. and on the R.H.S. square of 2 is set.

Solved Examples; -

1.	97^2	=	$97 - 3/ 3^2$	=	94/9	=	9409
2.	95^2	=	$95 - 5/ 5^2$	=	90/25	=	9025
3.	92^2	=	$92 - 8/ 8^2$	=	84/64	=	8464
4.	89^2	=	$89 - 11/ 11^2$	=	78 / 121	=	7921

Slash (/) is used here to separate the left-hand side and the right hand side of the number.

Exercise - 5

1.	96^2	=	
2.	85^2	=	
3.	87^2	=	
4.	81^2	=	
5.	79^2	=	
6.	77^2	=	
7.	84^2	=	
8.	93^2	=	
9.	91^2	=	
10.	99^2	=	

Real Magic

I am certain that you will experience thrill after learning and understanding these methods. You will find this magical. Also you will find this very easy to work with. Try to teach these methods to as many persons as you can.

Denominator ending with 9.

Find $\frac{73}{139}$ up to 5 places of decimals. Let us try to solve it first by conventional method :-

```
139)  730    (0 . 5 2 5 1 7
      695
     -----
       350
       278
      -----
        720
        695
      -----
         250
         139
      ------
          1110
           973
       ------
           137
```

You people are well verse with conventional method so I am skipping the explanation.

Now, Let us see the magical method :-

$$\frac{73}{139} = \frac{7.3}{13.9} \cong \frac{7.3}{14} = \begin{matrix} 0.\ 5\ 2\ 5\ 1\ 7 \\ \text{~~3 7 2 11~~} \end{matrix}$$

Check, whether two answers are same (?)

By conventional method our answer to 5 places of decimal is 0.52517.

By magical method also our answer is 0.52517.

There is no difference between the answers, however the procedure adopted in both the methods is different. One is more cumbersome than the other. Let me explain the steps.

Steps:-

1. 73 is divided by 139 (a digit ending with 9)
2. $\frac{73}{139}$ is reduced to $\frac{7.3}{13.9}$ or $\frac{7.3}{14}$
3. Start dividing 73 by 14.
4. Put the decimal point first, divide 73 by 14, 5 is Quotient and 3 is remainder, 5 is written after the decimal and 3 is written in front of 5 below it as shown.
5. Our next gross number is 35, divide 35 by 14. Quotient = 2 and Remainder = 7. Q = 2 is written after 5 and R = 7 before 2 (below it)
6. Our next gross number is 72, divide 72 by 14. Q = 5 and R = 2, Q = 5 is written after 2 and R = 2 before 5 (below it).
7. Our Next gross number = 25, divide 25 by 14. Quotient = 1 and remainder = 11. Q = 1 is written after 5 and R = 11 before 1 (below it).
8. We have already found answer up to four decimal places, our next dividend is 111, divide by 14. Quotient = 7, thus we have completed finding the answer up to five places of decimal.
9. Repeat the above steps if you want to find the values further.

You have learned the steps required to solve such kind of problems where the denominator ends with 9. Let us take some more examples to make our understanding clear.

Examples :-

1. <u>75</u> = <u>7.5</u> ≅ <u>75</u> = 0. 5 3 9 5 6 8- Answer

139	13.9	14	~~5 13 7 9 11~~ -	Remainders

2. $\frac{63}{149} = \frac{6.3}{14.9} \cong \frac{6.3}{15} =$ 0. 4 2 2 8 1 8 7 -Answer

~~3 4 12 2 13 11~~ -Remainders

3. $\frac{83}{189} = \frac{8.3}{19} =$ 0. 4 3 9 1 5 3 - Answer

~~7 17 2 10 6 8~~ - Remainders

Exercise - 6

1.	$\frac{76}{139}$	=	
2.	$\frac{64}{129}$	=	
3.	$\frac{1}{19}$	=	
4.	$\frac{1}{29}$	=	
5.	$\frac{63}{129}$	=	
6.	$\frac{43}{179}$	=	
7.	$\frac{83}{119}$	=	
8.	$\frac{76}{189}$	=	
9.	$\frac{53}{149}$	=	
10.	$\frac{57}{159}$	=	

Denominator digit ending with 8.

You will ask me a question, whether the process is applicable only if a denominator ends with 9. Answer is no. We can apply this technique to digits ends with 8, 7, 6 etc. but with slight change.

Let us see it for denominator ending with 8:-

+5+2+8+9

$\frac{73}{138} = \frac{7.3}{13.8} \cong \frac{7.3}{14} =$ 0. 5 2 8 9 8

~~3 12 12 10~~

In case of denominator digits ending with 8 (one less than 9) the steps are as follows :-

1. Placing of Remainder infront of Quotient remains same as explained in the case $\frac{73}{139}$ denominator digit ending with 9 .
2. In the Quotient digit 1 time (9 – 8= 1) of the Quotient digit is added at every step and divided by the divisor for finding out the answer.

As in this case we found our first Q1 = 5 and R1 = 3, Our gross dividend comes out to be 35 in which we added 5 to make it 40 then divided by 14. In the next step Q2 = 2 and R2 = 12 Our gross dividend at step 2 becomes 122 + Q2 = 124. Divide this by 14.

The procedure is repeated to find the solution to required number of decimal places.

Let us take some more examples so that we can understand it better :-

+4 +4 +6+4 +2

1. $\frac{75}{168} = \frac{7.5}{16.8} \cong \frac{7.5}{17} =$ 0.4 4 6 4 2 8

~~7 10 6 4 14~~

+4 +6 +6 +2

2. $\frac{83}{178} = \frac{8.3}{17.8} \cong \frac{8.3}{18} =$ 0.4 6 6 2 9

~~11 10 4 16~~

+1+6 +4 +8

3. $\frac{31}{188} = \frac{3.1}{18.8} \cong \frac{3.1}{19} =$ 0.1 6 4 8 9

~~12 8 16 16~~

Exercise - 7

1.	$\frac{78}{138}$	=	
2.	$\frac{74}{148}$	=	
3.	$\frac{63}{128}$	=	
4.	$\frac{51}{118}$	=	
5.	$\frac{56}{118}$	=	
6.	$\frac{49}{128}$	=	
7.	$\frac{83}{178}$	=	
8.	$\frac{79}{148}$	=	
9.	$\frac{32}{148}$	=	
10.	$\frac{37}{168}$	=	

Overcoming Problem with Tables

Addition and subtraction is quite fast and one does not find any difficulty with addition and subtraction. One can even do it with jet set speed. But when it comes to multiplication normally it is difficult.

Poor command over table makes it worse.

Probably you must be aware of the policies of British Empire, which they used so effectively to gain and retain power in India for more than 200 years. Probably you have recollected it, the policy is very famous and this can be stated as "Divide and Rule".

You may wonder why I am describing this policy here and what is its relation with multiplication. You do not have to wait long for understanding why I have talked about that policy here. Like British Empire you have to use that policy to fathom calculation related with tables. This policy can be applied in all sorts of calculation. So what I am suggesting? I am suggesting that whenever in difficulty break it and rule over it. How?

Example

$$17 \times 8 = ?$$

It's slightly difficult to write the answer directly. But if you apply divide and rule policy here you can do it mentally. How? The above multiplication can be broken as given below.

$$(10 + 7) \times 8 =$$
$$80 + 56 = 136$$

Now you can work with this very easily. And write the answer as 136.

Example

$19 \times 7 = ?$

You can also break them as follows:

$(10 + 9) \times 7 =$

$70 + 63 = 133$

Exercise - 8

S. No.	Problem	Answer
1.	17×8	
2.	16×6	
3.	12×9	
4.	18×7	
5.	19×9	
6.	16×8	
7.	14×9	
8.	17×7	
9.	18×9	
10.	19×8	
11.	14×8	
12.	16×9	
13.	15×8	
14.	13×9	
15.	17×6	

Exercise - 9

S. No.	Problem	Answer
1.	14×8	
2.	13×6	
3.	18×9	
4.	18×6	
5.	19×7	
6.	16×6	
7.	17×4	
8.	19×4	
9.	14×8	
10.	19×5	
11.	18×8	
12.	16×8	
13.	12×8	
14.	18×3	
15.	17×6	

Result Verification Technique

There are various methods of verifying the results obtained after completion of a mathematical operation. Vedic Mathematics offers a very simple method of result verification, which can be used in day-to-day activity with great ease.

Here we would use Navshesh

Navshesh means 'nine and its remainder '.

Finding Navshesh

Any number can be converted into a single digit (Navshesh) after adding the digits. Let me explain this with the help of examples.

Example

Find single digit equivalent for 32.

Just add 3 and 2. You will get 5

The single digit equivalent (Navshesh) is 5.

Example

Find single digit equivalent for 342

In this case you will be required to add them all

$3+4+2 \quad = \quad 9$

The single digit equivalent (Navshesh) is 9.

Example

Find single digit equivalent for – 732?

In this case you will be required to add them all

$-(7+3+2) \quad = \quad -12 \quad = \quad -(1+2) \quad = -3 \quad = (-3+9) = 6$

Incase of a negative number add 9 to it. The single digit equivalent (Navshesh) is 6.

After going through these examples you can find out single digit equivalent Navshesh of any large number.

Exercise - 10

Find Navshesh of the following numbers:

S. No.	Number	Navshesh
1.	362	
2.	3552	
3.	4999	
4.	5796	
5.	2643	
6.	8871	
7.	-7792	
8.	39495	
9.	23994	
10.	5299	
11.	63692	
12.	5439	
13.	21799	
14.	-91669	
15.	19993	

The Navshesh methodology gives you a quick way of verifying the results. How?
You can use this methodology in Addition, Subtraction and Multiplication and division to verify the results. This formula simply states that the **Navshesh remains unchanged**. In other words, Navshesh of the digits before operation and after operation will remain unchanged.

Example:

65 + 37 = 102	Problem Side: Navshesh of 65 is 2 and navshesh of 37 is 1 adding navshesh of of both the numbers gives **3**. Answer Side: Navshesh of 102 is **3**
873 – 439 = 434	Problem Side: Navshesh of 873 is 9 and navshesh of 439 is 7 Subtracting 7 from 9 gives **2**. Answer Side: Navshesh of 434 is **2**
106 × 108 = 11448	Problem Side: Navshesh of 106 is 7 and navshesh of 108 is 9 Multiplying 7 with 9 gives 63 whose Navshesh is 9. Answer Side: Navshesh of 11448 is **9**
512 ÷ 8 = 64	Problem Side: Navshesh of 512 is 8 and navshesh of 8 is 8 Dividing 8 with 8 gives 1 whose Navshesh is **1**.

	Answer Side: Navshesh of 64 is **1**

Exercise- 11

Find out whether the answers are correct:

S. No.	Problem	Yes/ No
1.	51 + 62 + 845 = 958	
2.	22 × 617 = 13884	
3.	3576 – 2412 – 162 = 2001	
4.	111104 ÷ 217 = 512	
5.	8311 – 1563 – 785 = 5963	
6.	324 × 216 = 69984	
7.	325 + 242 + 841 = 1208	
8.	8311 – 1563 – 785 = 5963	
9.	19548 ÷ 362 = 54	
10.	52 × 514 × 48 = 1282934	
11.	71028 ÷ 1973 = 36	
12.	258 + 142 + 84832 = 87564	
13.	147368 ÷ 338 = 536	

14.	56 × 64 × 62 = 222208	
15.	311 + 563 + 7851 = 8725	

Overcoming Problem with Percentage Calculation

You encounter percentage calculation at several places everyday. The biggest examples can be calculation of exact amount offered as discount while shopping.

The percentage calculation can be termed as difficult because the amount of calculation involved. My endeavor is to reduce the difficulty.

How to reduce the difficulty?
Before starting the work on this, let us analyse the way this operation is being carried out. Suppose I have to find out x% of Y then the operation will contain x multiplied by Y and whole divided by 100. This will give us the desired answer. It seems very simple. Then where is the difficulty?

The difficulty is Multiplication and division at the same time. Therefore, we suggest **10 % Methods.**

10 % method? What is this?

Can you find 10% of a number?

It's the easiest % one can find. If you can find 10% of a number then you are through. How?

Convert % into multiples of 10%.

Let us take an example.

Find 11% of 3365

$$11\% = 10\% + 1\%$$

$$= 10\% + \frac{10\%}{10}$$

$$= 336.5 + 33.65 = 370.15$$

What about 9%

$$9\% = 10\% - \frac{10\%}{10}$$

$$= 336.5 - 33.65$$

$$= 302.85$$

Exercise-12

A shopkeeper is offering the following discount on his products. Find how much you would be required to pay him if list price is following:

S. No.	List Price	Discount	You Pay
1.	365	10	
2.	550	11	
3.	999	9	
4.	579	15	
5.	643	16	
6.	887	20	
7.	779	21	
8.	495	19	
9.	399	6	
10.	299	31	
11.	369	22	
12.	439	33	
13.	1799	25	

14.	1669	12	
15.	1999	10	

Finding Cube Root of a Number by Visualisation

Finding cube root of a number is one of the very difficult tasks one can encounter with. The only method known to us till date is to find the factors of a number and then group the factors taking three at a time and take out one number from a group of three. Let us explain this with the help of an example:

Example

Find cube root of 1728

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

We have found the factors now. The next step is to make a group of taking three numbers together.

$$\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$$

Now if you take out one number from a group of three numbers then you will get

$$2 \times 2 \times 3 = 12$$

You have got your answer as 12

Example

Find cube root of 9261

3	9261
3	3087
3	1029
7	343
7	49
	7

We have found the factors now. The next step is to make a group of taking three numbers together.

$$\underline{3 \times 3 \times 3} \times \underline{7 \times 7 \times 7}$$

Now if you take out one number from a group of three numbers then you will get

$$3 \times 7 = 21$$

You have got your answer as 21

Finding factors of a number is a cumbersome task. This takes a lot of time.

In contrast Magical Methods provides you with a one-line answer and anybody can find cube root of a number mentally once he/she understands the technique.

Finding cube roots will require some background: -

Last digit

1^3	=	1	1
2^3	=	8	8
3^3	=	27	7
4^3	=	64	4
5^3	=	125	5
6^3	=	216	6
7^3	=	343	3
8^3	=	512	2
9^3	=	729	9

From the above illustration we can take out that last digit of 2^3 is 8, 3^3 is 7 and vice-versa. All other repeats itself.

Procedure of finding the cube root: -

1. Start from right and put a comma when three digits are over because a group of three digits will give you one digit as cube root. Four, five and six digit number will give you two digits as cube root.

Example:-
1. 9,261
2. 1,728
3. 32,768
4. 175,616

2. After putting the comma see the last digit of the number, compare that with table provided above. You get the last digit.
3. Now see the first group of numbers and ascertain cube of which number is less than the group. That number is your first digit.

4. You have thus found first digit and last digit.

Let us take an example:-

Example

- 9, 261

  2 1

Steps :-

1. Counting from last we put comma after 9.
2. By seeing the last digit we ascertain that last digit of cube root will be 1.
3. Now we see 9 and ascertain that 2^3 = 8 is less than 9 and 3^3 = 27 is more.
4. Our first digit thus comes to 2, and the answer is 21.

Example

32,768

3 2

1. By seeing last digit we find last digit of cube root is equal to 2.

2. By seeing 32 we put 3 as our first digit as 3^3 = 27 is less than 32 and 4^3 = 64 is more.

3. Our answer is 32.

Example

140,608

5 2

4. By seeing last digit we find last digit of cube root is equal to 2.

5. By seeing 140 we put 5 as our first digit as 5^3 = 125 is less than 140 and 6^3 = 216 is more.

6. Our answer is 52.

Example

438,976
7 6

7. By seeing last digit we find last digit of cube root is equal to 6.

8. By seeing 438 we put 7 as our first digit as 7^3 = 343 is less than 438 and 8^3 = 512 is more.

9. Our answer is 76.

Note :- This technique is valid for exact cubes only.
This is a good method of finding approximations.

Find Cube root of the following numbers

Exercise-13

1.	74088	=	------
2.	29791	=	------
3.	32768	=	------
4.	103823	=	------
5.	91125	=	------
6.	35937	=	------
7.	140608	=	------
8.	59319	=	------
9.	13824	=	------
10.	97336	=	------
11.	110592	=	------

12.	117649	=	------
13.	24389	=	------
14.	4913	=	------
15.	17576	=	------
16.	50653	=	------
17.	2744	=	------
18.	5832	=	------
19.	21952	=	------
20.	6859	=	------